# The chain rule

Differentiate a composition by following the inner change and the outer change.

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## 1 / The rule

Suppose g is differentiable at x and f is differentiable at g(x). For a function formed by putting g inside f:

$$\frac{d}{dx} f(g(x)) = f'(g(x))\, g'(x)$$

First differentiate the outer function while keeping its input unchanged. Then multiply by the derivative of the inner function.

## 2 / Example: a polynomial inside a power

$$y = (3x^2 + 1)^4$$

Inner function: g(x) = 3x² + 1, so g′(x) = 6x. Outer function: f(u) = u⁴, so f′(u) = 4u³.

$$y' = 4(3x^2 + 1)^3 \cdot 6x = 24x(3x^2 + 1)^3$$

## 3 / Example: a squared input to sine

$$y = \sin(x^2) \quad \Longrightarrow \quad y' = 2x\cos(x^2)$$

The cosine comes from differentiating sine. The factor 2x comes from differentiating the input x².

## Check your work

A common mistake is to stop after differentiating the outer function. Ask: “Did I also differentiate the inner input?”

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