Linear approximation

Use the tangent line near a known value to estimate a nearby function value.

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1 / The idea

If f is differentiable at a, its tangent line gives a local approximation:

$$f(x) \approx L(x) = f(a) + f'(a)(x - a)$$

The value f(a) supplies a known starting point; f′(a) supplies the slope. The estimate is intended for x close to a.

2 / Example: estimate √4.1

Choose f(x) = √x and a = 4, because √4 is known exactly.

$$f(4) = 2, \qquad f'(x) = \frac{1}{2\sqrt{x}}, \qquad f'(4) = \frac{1}{4}$$

$$\sqrt{4.1} \approx 2 + \frac{1}{4}(4.1 - 4) = 2.025$$

The square-root curve is concave down for x > 0, so its tangent line lies above the curve. This estimate is slightly too large.

3 / How close is close enough?

A small distance helps, but curvature also matters. If f has a continuous second derivative between a and x and |f″| ≤ M there, Taylor’s theorem gives:

$$|f(x) - L(x)| \le \frac{M}{2}|x - a|^2$$

For √x on [4, 4.1], the magnitude of f″(x) is at most 1/32, its value at x = 4. The error is therefore at most:

$$\frac{1}{2} \cdot \frac{1}{32} \cdot (0.1)^2 = 0.00015625$$

Use the approximation near the chosen point. Near x = 0, the square-root derivative becomes very large, so this particular starting point and bound do not apply.

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