The product rule

When both factors change, the derivative has a contribution from each one.

---

1 / Derivation from small changes

Let h(x) = u(x)v(x), with u and v differentiable at x. For a nonzero change Δx, write Δu = u(x + Δx) - u(x), and similarly for Δv.

$$\Delta h = (u + \Delta u)(v + \Delta v) - uv$$

$$\Delta h = u\,\Delta v + v\,\Delta u + \Delta u\,\Delta v$$

Divide by Δx and take the limit as Δx approaches zero:

$$\frac{\Delta h}{\Delta x} = u\frac{\Delta v}{\Delta x} + v\frac{\Delta u}{\Delta x} + \frac{\Delta u}{\Delta x}\Delta v$$

The last term tends to zero: Δu/Δx tends to u′(x), while Δv tends to zero. Therefore:

$$h'(x) = u(x)v'(x) + v(x)u'(x)$$

2 / Example: x² times sine

$$h(x) = x^2 \sin x$$

$$h'(x) = x^2 \cos x + 2x \sin x$$

The first term keeps x² and differentiates sin x. The second keeps sin x and differentiates x².

Check your work

The derivative of a product is generally not the product of the derivatives. Include both contributions.

This sample illustrates one possible PDF note format. It does not describe the website owner’s coursework or achievements.