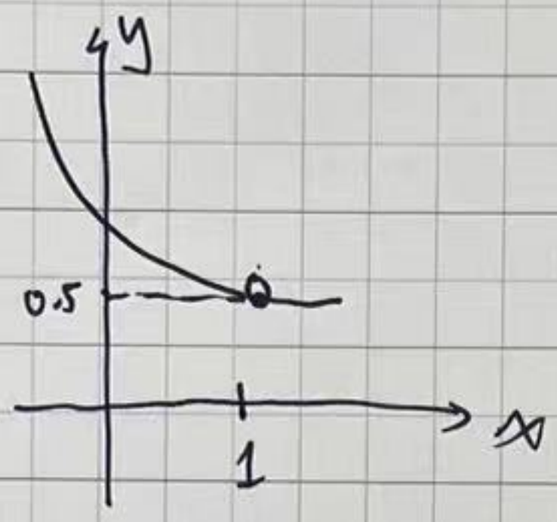
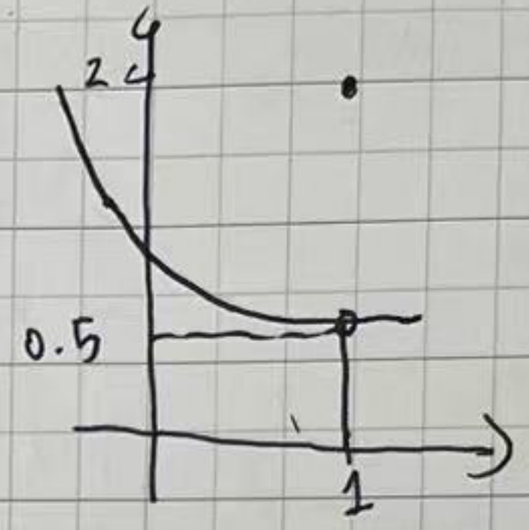
Limit.

① $f(x)=(x-1)/(x^2-1)$ is not defined when $x=1$

$$\lim_{x\to 1}\frac{x-1}{x^2-1}=0.5$$

②

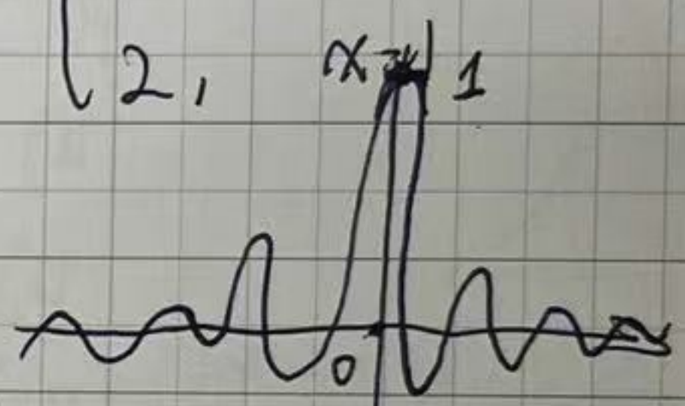




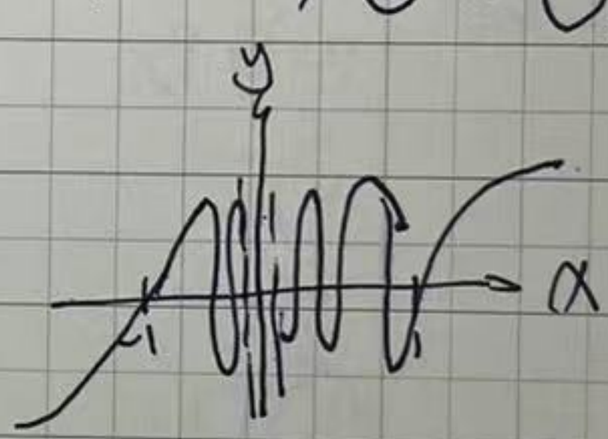
$f(x)$: $y=\dfrac{x-1}{x^2-1}$, $x\neq 1$

$g(x)\begin{cases} y=\dfrac{x-1}{x^2-1}, & x\neq 1 \\ 2, & x=1 \end{cases}$

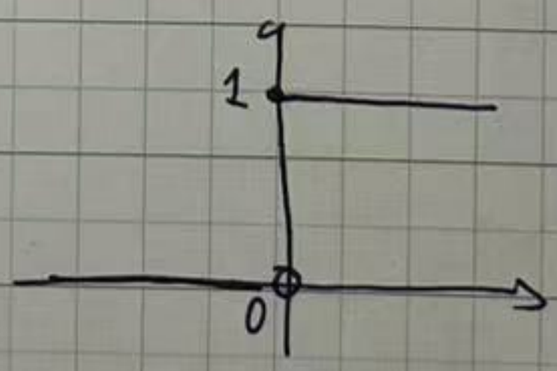
$\lim_{x\to 0}\dfrac{\sin x}{x}=1$ $f(x)=\dfrac{\sin x}{x}$:



$\lim_{x\to 0}\sin\frac{\pi}{x}$ = no exist.



$\lim_{t\to 0^-}H(t)=0$ & $\lim_{t\to 0^+}H(t)=1$.



$\therefore \lim_{t\to 0} H(t)$ does not exist.

Summary

Vertical Asymptote.

$\lim_{x\to a} f(x)=-\infty/\infty$

↳ $\lim_{x\to a} f(x)=L$ only if $\lim_{x\to a^-} f(x)=L$ & $\lim_{x\to a^+} f(x)=L$

$\lim_{x\to a}\boxed{\infty}\Rightarrow$ no limit. { "limit of $f(x)$, as x approaches a, is infinity" / "$f(x)$ increase w/o bound as x approaches a"

15. $\lim_{x\to a} f(x)$ is continuous at a. only if $\lim_{x\to a} f(x) = f(a)$

16. If f & g are continuous at a. (C is constant)

then $f \pm g$

cf

fg

$\frac{f}{g}$ if $g \neq 0$.

are all continuous at a.

17. $\mathbb{R} = (-\infty, \infty)$ for domain

1. $\lim_{x\to a} [f(x) \pm g(x)] = \lim_{x\to a} f(x) \pm \lim_{x\to a} g(x)$

2. $\lim_{x\to a} [c \cdot f(x)] = c \lim_{x\to a} f(x)$

3. $\lim_{x\to a} \frac{f(x)}{g(x)} = \frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)}$ if $\lim_{x\to a} g(x) \neq 0$.

4. $\lim_{x\to a} [f(x)]^n = [\lim_{x\to a} f(x)]^n$

5. $\lim_{x\to a} c = c$ 6. $\lim_{x\to a} x = a$.

9. $\lim_{x\to a} x^n = a^n$.

10. $\lim_{x\to a} f(x) = f(a)$ if only if f is polynominal or a rational function and a is in the domain

11. $\lim_{x\to a} f(x) = L$ if & only if $\lim_{x\to a^-} f(x) = L = \lim_{x\to a^+} f(x)$

12. $[\![4]\!] = 4$, $[\![4.8]\!] = 4$

13. If $f(x) \leq g(x)$, then $\lim_{x\to a} f(x) \leq \lim_{x\to a} g(x)$.

14. If $f(x) \leq g(x) \leq h(x)$, and $\lim_{x\to a} f(x) = \lim_{x\to a} h(x) = L$. then $\lim_{x\to a} g(x) = L$.

Summary

ie. $\lim_{x\to 0} x^2 \sin\frac{1}{x} = 0$.

$\therefore \lim_{x\to 0} x^2 \sin\frac{1}{x} = 0$.

$-1 \leq \sin\frac{1}{x} \leq 1$.

$-x^2 \leq \sin\frac{1}{x} \leq x^2$.

$\lim_{x\to 0} -x^2 = 0$ $\lim_{x\to x^2} = 0$.

$\therefore \lim_{x\to 0} \sin\frac{1}{x} = 0$.

fog

f-composed of g

i.e. $|x^2+1|$ continuous.

$\sin(x)$ continuous.

⇓

$\sin(x^2+1)$

continuous.

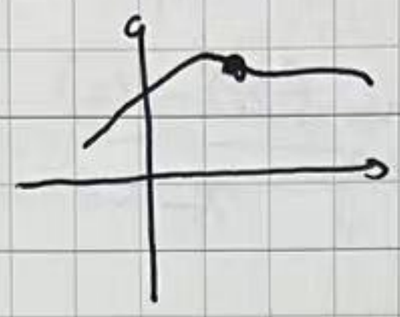
All polynomial are continuous $(-\infty, \infty)$

Rational ~~Continuous~~ can be discontinuous.

IVT · a is in $[c,b]$ then if $[c,b]$ is continuous, there must be such a.

Limit - Continuity.

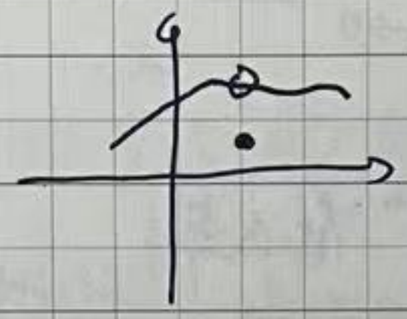
① f is continuous at $x=a$ if $\lim_{x\to a} f(x) = f(a)$.



② f is not continuous at $x=a$ which here $\lim_{x\to a} f(x)$ exists but $f(a)$ does not



③ f is not continuous at $x=a$ which here $\lim_{x\to a} f(x)$ exists and $f(a)$ exists but $\neq$.



$\lim_{x\to a^+} \neq \lim_{x\to a^-} \therefore \lim_{x\to a}$ DNE

Limit law for division.

① $\lim_{x\to a} f(x) = L \quad \lim_{x\to a} g(x) = M$.

↳ $M \neq 0 \Rightarrow \frac{L}{M}$.

↳ $M = 0, L \neq 0 \Rightarrow$ DNE.

↳ $M = 0, L = 0 \Rightarrow$ might exist.

② $\lim_{x\to 1} \frac{(x-1)(x+3)}{(x-1)(x-2)} \Rightarrow$ Not defined when $x=1$.

⇓

$\lim_{x\to 1} \frac{x+3}{x-2}$ defined when $x=1$

③ $\lim_{x\to 0^+} \frac{1}{x} = +\infty$.

④ when 0^+ use 0.1

when 0^- use -0.1

Summary