$(b-x)(b^{n-1}+b^{n-2}x+b^{n-3}x^2+b^{n-4}x^3+\cdots bx^{n-2}+x^{n-1})=$

$b^n+\cancel{b^{n-1}x}+\cancel{b^{n-2}x^2}+\cdots b^2x^{n-2}+bx^{n-1}-\cancel{bx^{n-1}}-\cancel{b^{n-2}x^2}-\cdots b^2x^{n-2}-bx^{n-1}-x^n$

$\therefore = b^n - x^n.$

Power Rule.

$f(x)=x^n \Rightarrow$ find $f'(x)=?$

$f'(x)=\frac{f(b)-f(x)}{b-x}=\lim_{b\to x}\frac{b^n-x^n}{b-x}.$

$=\lim_{b\to x}\frac{(b-x)(b^{n-1}+b^{n-2}x+b^{n-3}x^2+b^{n-4}x^3\cdots bx^{n-2}+x^{n-1})}{b-x}$

$\because$ polynomial

$\therefore b=x \Rightarrow = x^{n-1}+x^{n-2}\cdot x+x^{n-3}x^2+x^{n-4}x^3+\cdots+xx^{n-2}+x^{n-1}$

$=(x^{n-1}+x^{n-1}+x^{n-1}+\cdots x^{n-1}+x^{n-1})$ (n)

$=n(x^{n-1})$ $\Rightarrow n$ is positive integer.

$f(x)=4x+3.$

$f'(5)=4\times5+3.$

$f(5)=$ Can't determined.

Tangent line at one point is $4x+3$.

$y=f(5)=4\times5+3.$

$f'(5)=4.$ (slope).

Notation:

$\frac{dy}{dx} \Leftarrow \frac{\Delta y}{\Delta x}(\Delta x\to 0).$ or $\frac{d}{dx}f(x).$

↳ limit of difference.

Summary

second: $f^{(2)}(x)=f''(x)=\frac{d^2f}{dx^2}$

↳ $f''(x)$ tell us is the rate of change increasing or decreasing

(+) (−)

$$(b-x)(b^{n-1}+b^{n-2}x+b^{n-3}x^2+b^{n-4}x^3+\cdots+bx^{n-2}+x^{n-1})=$$

$$b^n+b^{n-1}x+b^{n-2}x^2+\cdots+b^2x^{n-2}+bx^{n-1}-xb^{n-1}-b^{n-2}x^2-\cdots-b^2x^{n-2}-bx^{n-1}-x^n$$

$$\therefore = b^n - x^n.$$

Power Rule.

$f(x)=x^n \Rightarrow$ find $f'(x)=?$

$$f'(x)=\frac{f(b)-f(x)}{b-x}=\lim_{b\to x}\frac{b^n-x^n}{b-x}.$$

$$=\lim_{b\to x}\frac{(b-x)(b^{n-1}+b^{n-2}x+b^{n-3}x^2+b^{n-4}x^3+\cdots+bx^{n-2}+x^{n-1})}{b-x}$$

$\because$ polynomial

$\therefore b=x \Rightarrow = x^{n-1}+x^{n-2}\cdot x+x^{n-3}x^2+x^{n-4}x^3+\cdots+xx^{n-2}+x^{n-1}$

$$=(x^{n-1}+x^{n-1}+x^{n-1}+\cdots+x^{n-1}+x^{n-1})$$

$$=n(x^{n-1}) \Rightarrow n \text{ is positive integer.}$$

$y=$

$f(x)=4x+3.$

$f'(5)=4\times5+3.$

$f(5)=$ Can't determined.

Tangent line at one point is $4x+3$.

$f(5)=4\times5+3.$

$f'(5)=4.$ (slope).

Notation:

$\frac{dy}{dx} \Leftarrow \frac{\Delta y}{\Delta x}\ (\Delta x\to 0).$ or $\frac{d}{dx}f(x).$

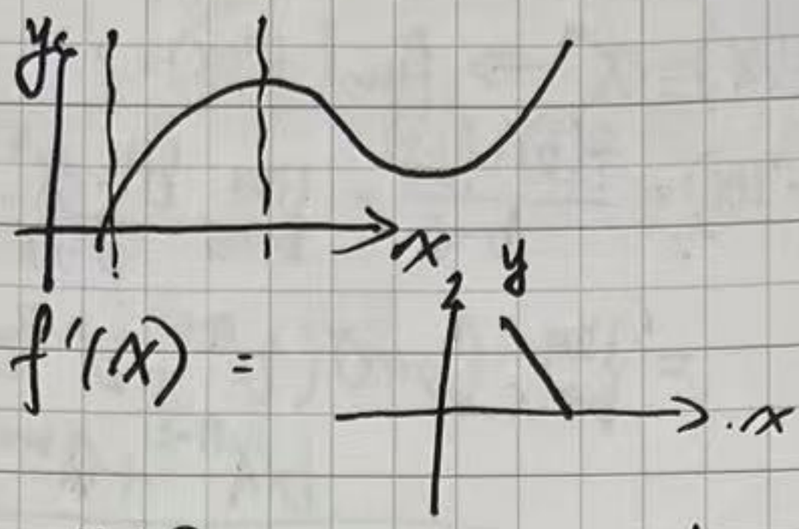
↳ limit of difference.

Summary

Secouel: $f^{(2)}(x)=f''(x)=\frac{d^2f}{dx^2}$

↳ $f''(x)$ tell us is the rate of change increasing or decreasing (+) (−)

Secoud derivative.



f''(x) : negative. ⇒ decreasing of aroc / negative slope of f'(x)

Trig derivative. (令 x=0 for 便幸捷).

① Sine derivative.

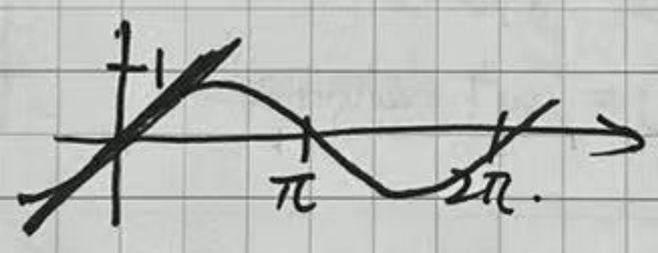
$$\lim_{\Delta x \to 0} \frac{\sin(x+\Delta x) - \sin x}{\Delta x} = \lim_{\Delta x \to 0} \frac{\sin x \cos \Delta x + \cos x \sin \Delta x - \sin x}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \sin x \left(\frac{\cos x - 1}{\Delta x}\right) + \cos x \left(\frac{\sin \Delta x}{\Delta x}\right)$$

$\sin x \cdot \cos' x +$
$\cos x \cdot \sin' x$
$= \sin x \cdot 0 + \cos x \cdot 1$
$= \cos\theta \cdot 1$
$= \cos\theta$
$\therefore \sin'\theta = \cos\theta$

↳ at $x=0$: $\frac{d}{dx}\sin(x)\Big|_{x=0} = \lim_{\Delta x \to 0} \frac{\sin(\Delta x)}{\Delta x} = 1$

measured in RADIANS ONLY!



$\therefore \sin'(0) = 1$



② Cosine derivative.

$$\lim_{\Delta x \to 0} \frac{\cos(x+\Delta x) - \cos x}{\Delta x} = \cancel{\cos}$$

↳ at $x=0$: $\frac{d}{dx}\cos(x)\Big|_{x=0} = \lim_{\Delta x \to 0} \frac{\cos(\Delta x) - 1}{\Delta x} = 0$.



$\therefore \cos'(0) = 0$

$\therefore \cos\theta = -\sin\theta$

Summary

let $\Delta x = \theta$

① Cosine derivative (algebraic).

$$\lim_{\theta \to 0} \frac{\cos\theta - 1}{\theta}$$

$$\because \frac{\cos\theta - 1}{\theta} = \frac{1-\cos\theta}{\theta} = \frac{1-\cos\theta}{\theta} \cdot \frac{1+\cos\theta}{1+\cos\theta}$$

$$= \frac{1-\cos^2\theta}{\theta(1+\cos\theta)} = \frac{\sin^2\theta}{\theta(1+\cos\theta)}.$$

$$= \frac{\sin\theta}{\theta} \cdot \frac{\sin\theta}{1+\cos\theta}.$$

$$\therefore \lim_{\theta \to 0} \frac{\sin\theta}{\theta} \cdot \frac{\sin\theta}{1+\cos\theta} = \sin'(0) \cdot \frac{\sin 0}{1+\cos 0} = 1 \cdot \frac{0}{2} = 0.$$

③ Cosine derivative.

$$\lim_{\Delta x \to 0} \cos' x = \frac{\cos(x+\Delta x) - \cos x}{\Delta x} = \frac{\cos x \cos\Delta x - \sin x \sin\Delta x - \cos x}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \cos x \left(\frac{\cos\Delta x - 1}{\Delta x}\right) - \sin x \left(\frac{\sin\Delta x}{\Delta x}\right).$$

$$= \cos x \cdot \cos' 0 - \sin x \sin' 0.$$

$$= \cos x \cdot 0 - \sin x \cdot 1$$

$$= -\sin x.$$

$$\therefore \cos' x = -\sin x.$$

1.2. Find $h(x) = \sin x + \sqrt{3}\cos x$ for $h'(x) = 0$.

$$h'(x) = \frac{d}{dx}(\sin x) + \frac{d}{dx}(\sqrt{3}\cos x)$$

$$= \cos x - \sqrt{3}\sin x.$$

$$\therefore \cos x = \sqrt{3}\sin x$$

$$\therefore x = \frac{\pi}{6} + k\pi.$$

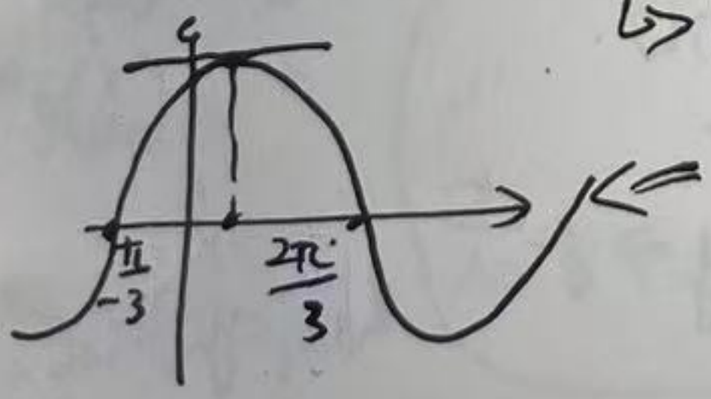
Summary

OR.

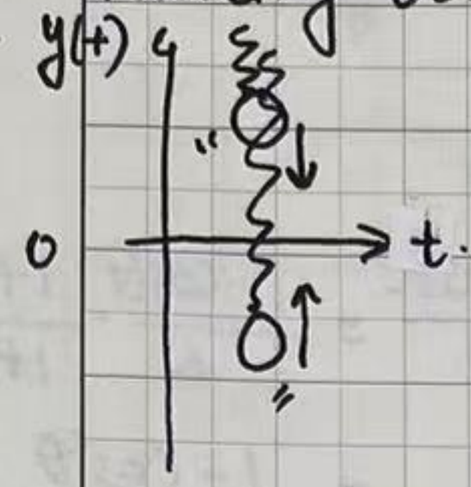
↳ $h(x) = 2\left(\frac{1}{2}\sin x + \frac{\sqrt{3}}{2}\cos x\right)$

$$= 2\left(\cos\frac{\pi}{3}\sin x + \sin\frac{\pi}{3}\cos x\right)$$

$$= 2\sin\left(\frac{\pi}{3} + x\right)$$



Modeling Oscillation.



$\because F=ma.$

$F=m\cdot y''(t).$ ①

$\because F=kx.$

$\begin{cases} x>0, F<0. \\ x<0, F>0. \end{cases}$

$\therefore F=-k\cdot y(t)$ ②.

$\therefore$ ①&②: $m\cdot y''(t)=-ky(t)$. Differential Equation

$\because \sin' x=\cos x.$ $\quad \because \cos' x=-\sin x.$

$\cos' x=-\sin x.$ $\quad -\sin x=-\cos x.$

$\sin'' x=-\sin x.$ $\quad \cos'' x=-\cos x.$

$\therefore m\cdot y''(t)=-ky(t)$ is similar.

$\therefore y''(t)=-\frac{k}{m}y(t).$

Solve: let $w^2=\frac{k}{m}. \Rightarrow w=\sqrt{\frac{k}{m}}.$

let $v(t)=y'(t)$ 降求导次数.

$\therefore \begin{cases} y'=t. \\ t'=-w^2 y. \end{cases} \Rightarrow \begin{cases} y'=0\cdot y+1\cdot t. \\ t'=-w^2 y+0\cdot t. \end{cases}$

$\Rightarrow M=\begin{bmatrix} y' \\ v' \end{bmatrix}=\begin{bmatrix} 0 & 1 \\ -w^2 & 0 \end{bmatrix}\begin{bmatrix} y \\ t \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$\det(M-\lambda I)=0 \Rightarrow \begin{bmatrix} y' \\ v' \end{bmatrix}=\begin{bmatrix} -\lambda & 1 \\ -w^2 & -\lambda \end{bmatrix}\begin{bmatrix} y \\ t \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \end{bmatrix}$

~~det(M-λI)~~

$\Rightarrow \therefore \lambda^2+w^2=0.$

$\lambda=-w^2.$

特征值: $\lambda_1=iw$, $\lambda_2=-iw.$

代入: ∵由矩阵行 $-\lambda y+t=0.$

$t=-\lambda y \Rightarrow y=\frac{t}{-\lambda}$

$\vec{u}_1=\begin{bmatrix} 1 \\ iw \end{bmatrix}$ $\vec{u}_2=\begin{bmatrix} 1 \\ -iw \end{bmatrix}$

注意:

$X(t)=C_1\begin{pmatrix} 1 \\ iw \end{pmatrix}e^{iwt}+C_2\begin{pmatrix} 1 \\ -iw \end{pmatrix}e^{-iwt}$

去虚部: $y(t)=C_1e^{iwt}+C_2e^{iwt}.$

OR

$y(t)=A\sin\left(\sqrt{\frac{k}{m}}t\right)+B\cos\left(\sqrt{\frac{k}{m}}t\right)$

∵ $y=\sin(wt)$

$y'=w\cos wt$

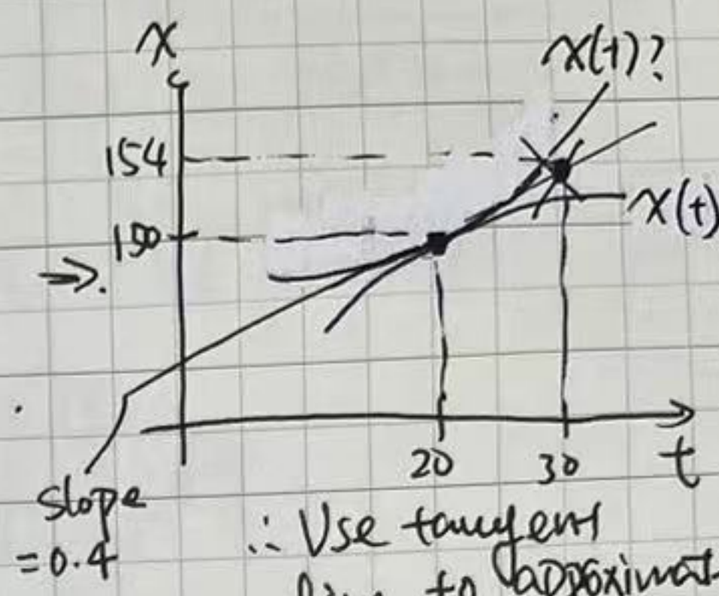
$y''=-w^2\sin wt.$

Summary:

1. Differentiation.

linear Approximation.

E.g. 150 m at $t=20s$, $v=0.4 m/s$. ⟶ ? m at $t=30s$, $V=?\ m/s$.



⇒ ∴ Use tangent line to approximate.
(Velocity same from ~~$t=20$~~ $t=20$ to $t=30$).

$\frac{x(30)-x(20)}{30-20}$ = slope at $t=20$.

$$\frac{g(x)-g(x_0)}{x-x_0} = g'(x_0)$$

$\therefore g(x)-g(x_0) \approx g'(x_0)(x-x_0) \Rightarrow x$ is near x_0.

$g(x) \approx g'(x_0)(x-x_0)+g(x_0)$

OR.

let slope between x & x = $\frac{\Delta g}{\Delta x}$.

$\therefore m = \lim_{\Delta x \to 0} \frac{\Delta g}{\Delta x} = \frac{d}{dx} g(x)$.

$\Rightarrow \Delta f \approx \left.\frac{df}{dx}\right|_{x=a} \cdot \Delta x$.

E.g. Find $3.97^{2.5}$.

Use $4^{2.5}$ to app bc derivative of 3.97^x is complex.

$\therefore \frac{3.97^{2.5}-4^{2.5}}{3.97-4} = 2.5 \times 4^{1.5}$

$3.97^{2.5} - 4^{\frac{5}{2}} = 2.5 \times 4^{\frac{3}{2}} \times (-0.03)$.

$3.97^{2.5} = 2.5 \times 8 \times (-0.03) + 32$.

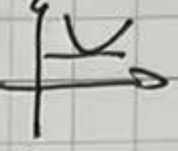
$= 20 \times (-0.03) + 32$.

$= -0.6 + 32$.

≈ 31.4.

Summary

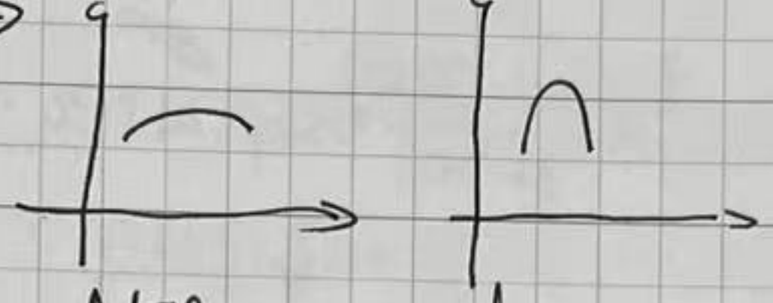
① Is the approximate line above or below the curve?

$y = f(x) \Rightarrow f''(x) > 0 \Rightarrow$ 

$\Rightarrow f''(x) < 0 \Rightarrow$ 

② More accurate?

↳



More accurate.

less accurate.

Eg. Boat (position, velocity, acceleration).

change of velocity the greatest = acc the greatest.

$\therefore f''(x)$ the greatest.

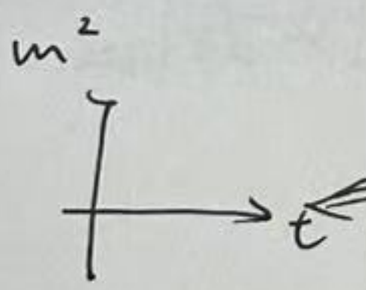
Prof. Why $h'(x) \neq f'(x) \cdot g'(x)$ if $h(x) = f(x)g(x)$.

let input x be t & output of $f(t)$ & $g(t)$ be m.

$\therefore h(x) = f(x)g(x)$ $\quad f'(t) = m/s \quad g'(t) = m/s$

$m^2 = m \cdot m.$ $\quad \therefore f'(t) \cdot g'(t) = m^2/s^2.$

$\therefore h'(t) = m^2/s.$



$\therefore m^2/s \neq m^2/s^2.$

$\therefore h'(t) \neq f'(t) \cdot g(t)$

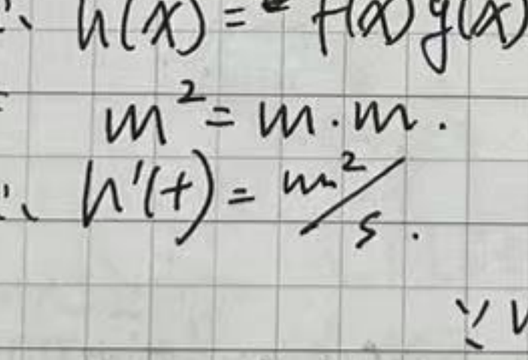
Summary

* ①: $\Delta A_{above} = 2\Delta t \cdot 50.$

$\therefore \frac{\Delta A_{above}}{\Delta t} = 100$

$\therefore h'(t) = f'(t)g(t) + g'(t)f(t)$

What is the $h'(t)$ actually cal as?



Find the rate of change of the Area?

① Area rate of change above +
② Area rate of change to right =
Total area roc.

① $50m \times 2\,m/s = 100\,m^2/s$:
$f(3) \times g'(3)$

② $30m \times 4\,m/s = 120\,m^2/s$
$g(3) \times f'(3)$

$\therefore$ Total $= 100 + 120 = 220\,m^2/s$

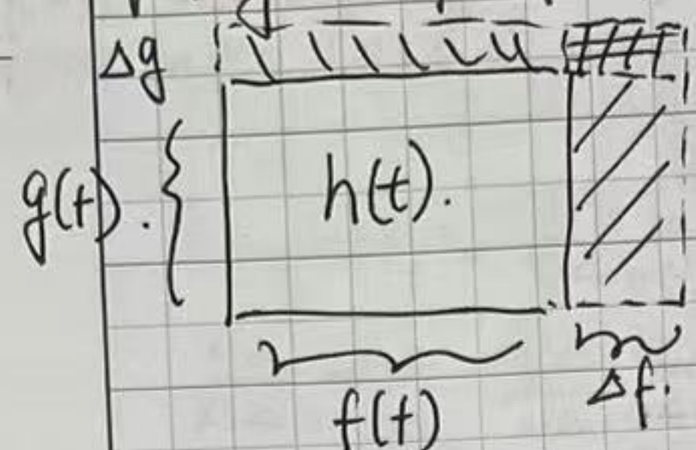
Product rule only applies to $f(x), g(x), t(x), \dots$

* NOT $f(10), g(5) \dots$

* $\frac{dx}{du} = \frac{d}{du}(x)$

$= \frac{d}{du}(f(u))$

* Algebra proof of Product Rule:



$\because \Delta h$ = new h - old h.

$\therefore \Delta h = (f+\Delta f)(g+\Delta g) - fg.$

$\therefore \Delta h = f\Delta g + \Delta f \cdot g + \Delta f \Delta g + fg$ (the $\Delta f \Delta g$ term is marked with a ?)

$\therefore \frac{\Delta h}{\Delta t} = f \cdot \frac{\Delta g}{\Delta t} + \frac{\Delta f}{\Delta t} \cdot g + \frac{\Delta f}{\Delta t} \cdot \frac{\Delta g}{\Delta t} \cdot \Delta t$

As $\Delta t \to 0$: $h' = f \cdot g' + f' \cdot g + f' \cdot g' \cdot 0.$

$\therefore h' = f \cdot g' + f' g.$

① Chain rule.

1. E.g. x = meters, u = seconds, t = minutes.

let $x = f(u) = 2u + 100 \Rightarrow \frac{dx}{du} = 2\ m/s$

$\because u = 60t \Rightarrow \frac{du}{dt} = 60\ s/min$

$\therefore x = g(u) = 120t + 100 \Rightarrow \frac{dx}{dt} = 120\ m/min.$

↳ $\frac{dx}{dt} = \frac{dx}{du} \cdot \frac{du}{dt}$ ✓

$120\ m/min = 2\ m/s \cdot 60\ s/min.$

2. $h(x) = f(g(x))$

$\because$ let $g(2) = 9,\ f(9) = 5,\ g'(2) = 3,\ f'(9) = 4.$

$\therefore h(2) = 5 \Rightarrow$ Find $h'(2)$

① $\frac{g(2+0.01) - g(2)}{0.01} = g'(2)$

$g(2.01) - 9 = 3 \times 0.01.$

$g(2.01) = 0.03 + 9 = 9.03.$

→ ② $\frac{f(9.03) - f(9)}{0.03} = f'(9)$

$f(9.03) - 5 = 4 \times 0.03.$

$f(9.03) = 0.12 + 5 = 5.12.$

→ ③ $\frac{h(2+0.01) - h(2)}{0.01} = h'(2)$

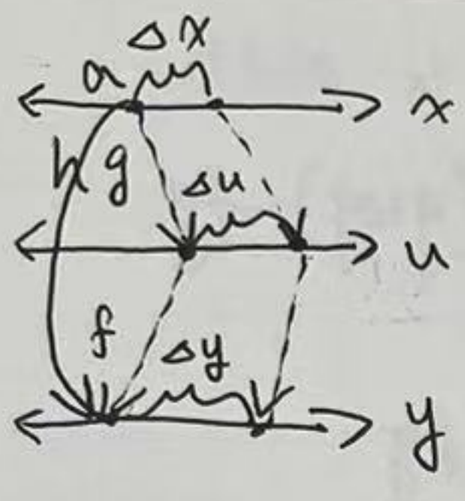
$\frac{0.12}{0.01} = 12.$

① $\frac{\Delta g}{\Delta x} = g'(2)$

$\Delta g = 3\Delta x \Rightarrow$ ② $\frac{\Delta f}{\Delta g} = f'(9)$

$\Delta f = 4\Delta g \Rightarrow \therefore 3 \times \Delta x \times 4 = 12\Delta x \Rightarrow \frac{\Delta h}{\Delta x} = 12.$ ✓ ③ $\Delta h = \Delta f.$

Summary



Deriving the chain rule.

$h(x) = f(g(x)) = f(u)$ where $u = g(x)$.

$\therefore h(x) = y$.

$\therefore \frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$ ✓

$= \lim_{\Delta u \to 0} \frac{\Delta y}{\Delta u} \cdot \lim_{\Delta x \to 0} \frac{\Delta u}{\Delta x}$.

($\Delta x \to 0$ then $\Delta u \to 0$) → $\frac{dy}{du}$

$\lim_{\Delta x \to 0} \frac{\Delta u}{\Delta x} = \frac{du}{dx}$

* Δu cancel but "du" can't

$\therefore \Rightarrow \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$

$$\boxed{* \left.\frac{dy}{dx}\right|_{x=a} = \left.\frac{dy}{du}\right|_{u=g(a)} \cdot \left.\frac{du}{dx}\right|_{x=a}.}$$

$$\boxed{h'(a) = f'(g(a)) \cdot g'(a)}$$

E.g. 1. Chain rule & 2. Quotient rule.

for $h(x) = \frac{f(x)}{g(x)} = f(x) \cdot [\text{reciprocal of } (r)]\, g(x)$

$\therefore h(x) = f(x) \cdot r g(x)$.

$\Rightarrow$ chain rule: $h'(x) = f(x) \cdot (rg(x))' + f'(x) \cdot (rg(x))$

$= f(x) \cdot \left(\frac{1}{g(x)}\right)' + f'(x) \cdot \left(\frac{1}{g(x)}\right)$

$= f(x) \cdot \left(-\frac{1}{g(x)^2}\right) \cdot g'(x) + \frac{f'(x)}{g(x)}$.

$= -\frac{f(x) \cdot g'(x)}{g(x)^2} + \frac{f'(x) \cdot g(x)}{g(x)^2}$

$= \frac{f'(x) \cdot g(x) - f(x) \cdot g'(x)}{g(x)^2}$.

= Quotient rule method.

* Chain rule ⇔ Quotient rule

Summary

Quorient Rule.

*Find h'
*known: $h=\frac{f}{g}$.

Proof 1:

$h\boxed{\frac{f}{g}}$

$(h+\Delta h)(g+\Delta g)-hg=\Delta f$.

$hg+h\cdot\Delta g+\Delta h\cdot g+\Delta h\cdot\Delta g-hg=\Delta f$.

$\frac{\Delta f}{\Delta t}=h\cdot\frac{\Delta g}{\Delta t}+\frac{\Delta h}{\Delta t}\cdot g+\frac{\Delta h}{\Delta t}\cdot\frac{\Delta g}{\Delta t}\cdot\Delta t$.

$\Delta t\to 0: f'=h\cdot g'+h'\cdot g+h'\cdot g'\cdot 0$.

$f'=h\cdot g'+h'\cdot g$

$\because h=\frac{f}{g}$.

$\therefore f'=\frac{f}{g}\cdot g'+h'\cdot g$

$f'-\frac{fg'}{g}=h'\cdot g$.

$\frac{f'}{g}-\frac{fg'}{g}\cdot\frac{1}{g}=h'$

$\Rightarrow \therefore h'=\frac{f'\cdot g}{g^2}-\frac{fg'}{g^2}$

$=\frac{f'\cdot g-fg'}{g^2}$

Proof 2: $\because h=\frac{f}{g}$. & From product rule $\Delta h=$ new $h-$ old h).

$\therefore \Delta h=\frac{f+\Delta f}{g+\Delta g}-\frac{f}{g}=\frac{fg+\Delta f\cdot g-(fg+f\Delta g)}{g(g+\Delta g)}=\frac{\cancel{fg}+\Delta f\cdot g-\cancel{fg}-f\Delta g}{g(g+\Delta g)}$

$=\frac{\Delta f\cdot g-f\cdot\Delta g}{g(g+\Delta g)}=\frac{\Delta f\cdot g-f\cdot\Delta g}{g^2+\Delta g\cdot g}$.

$\therefore \frac{\Delta h}{\Delta t}=\frac{\frac{\Delta f}{\Delta t}\cdot g-f\cdot\frac{\Delta g}{\Delta t}}{g^2+\Delta g\cdot g}$

$\Delta t\to 0 \Rightarrow h'=\frac{f'\cdot g-f\cdot g'}{g^2}$

(* $g+\Delta g=g(t+\Delta t)$).

$\therefore \Delta t\to 0\Rightarrow g(t+\Delta t)\to g$.

$\therefore g+\Delta g\to g$. $\therefore \Delta g\to 0$.

Summary * $h=\frac{f}{g}$. $\Rightarrow$ when $f\uparrow$, g & stays same. $\Rightarrow h\uparrow$ & $f\uparrow\Rightarrow h'$ & f' same sign.

$\Rightarrow$ when $g\uparrow$, f stays same $\Rightarrow g\uparrow, h\downarrow\Rightarrow h'$ & g' opposite sign.

Using quorient rule for $x^{-n}\Rightarrow \frac{d}{dx}\frac{1}{x^n}=-n\cdot x^{n-1}$.

$\because \frac{d}{dx}x^{-n}=-n\cdot x^{n-1}$.

$\therefore$ Power rule is for any INTEGER (for now)

Implicit Differentiation.

E.g. $x^2+y^2=25$. ← Implicit description of the two functions

$\Downarrow$

$x^2+y(x)^2=25$.

$\therefore \frac{d}{dx}(x^2+y(x)^2)=\frac{d}{dx}25$.

$\therefore 2x+2y(x)\cdot\frac{dy}{dx}=0$.

$\frac{dy}{dx}=\frac{-2x}{2y(x)}=-\frac{x}{y}$.

↳ ① If $y=0$, $\frac{dy}{dx}$ = undefined. $\Rightarrow$ ○|

② If $x=0$, $\frac{dy}{dx}=0$. $\Rightarrow$ ○

Power rule.
⇓ Apply to rational power.

* let $y=x^{m/n}$.

$\therefore \frac{d}{dx}y^n=\frac{d}{dx}x^m$

$\therefore n\cdot y^{n-1}\cdot\frac{dy}{dx}=m\cdot x^{m-1}$.

$\frac{dy}{dx}=\frac{m\cdot x^{m-1}}{n\cdot y^{n-1}}=\frac{m}{n}\cdot\frac{x^{m-1}}{y^{n-1}}$

$=\frac{m}{n}\cdot\frac{x^{m-1}}{x^{\frac{m}{n}(n-1)}}=\frac{m}{n}\cdot\frac{x^{m-1}}{x^{m-\frac{m}{n}}}=\frac{m}{n}\cdot x^{m-1-m+\frac{m}{n}}$

$=\frac{m}{n}\cdot x^{\frac{m}{n}-1}$.

$\therefore y=x^{m/n}$

$\Downarrow$

$\frac{d}{dx}y^2=2y\cdot\frac{dy}{dx}$.

Summary

Def of Derivative.

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

Limit.

① If $p(x)=(x-a)^{\text{can}\,5}\cdot s(x)$ & $q(x)=(x-a)^7\cdot r(x)$.

then $\lim_{x\to a}\frac{p(x)}{q(x)}=\frac{(x-a)^5\cdot s(x)}{(x-a)^7\cdot r(x)}$ where $x\to a$.

↳ $\lim_{x\to a}\frac{1}{(x-a)^2}\cdot\lim_{x\to a}\frac{s(x)}{r(x)}\Rightarrow$ [① $\frac{1}{(\to a^+-a)^2}\approx\frac{1}{(a+0.01-a)^2}\approx\frac{1}{\underbrace{0.01^2}_{\to 0}}\Rightarrow\infty$]

$\to\infty\cdot\to$ constant $\Leftarrow$ { [② $\frac{1}{(\to a^--a)^2}\approx\frac{1}{(a-0.01-a)^2}\approx\frac{1}{(-0.01)^2}\to\infty$] [③ $\frac{(x-b)^n}{(x-c)^m}\Rightarrow\frac{(a-b)^4}{(a-c)^4}\Rightarrow$ non-zero constant].

↓

DNE.

* If $\to 0\cdot\to$ constant, then $=0$.

If $\not\to 1\cdot\to$ constant, then $=$ constant.

② $f(x)=\frac{x-\sqrt[3]{x}+\cos x}{x^2-4}$ $\Rightarrow$ { $f(-3)=-0.51$) $x=-3\sim-1.5$ contains -2.
$f(-1.5)=0.16$) y changes 0. (from $-\to+$)
$f(1.5)=-0.24$ ✓
$f(2.25)=0.29$) $x=1.5\sim2.25$ contains 2.

↳ Num of roots at least?

$\because f(x)=\frac{x-\sqrt[3]{x}+\cos x}{\underbrace{(x+2)(x-2)}}\Rightarrow\therefore x\neq-2, x\neq 2$.

$\therefore$ at least 1 root.

Summary

$\therefore$ Derive ① & ② and plug 1.

$\therefore$ when $=$

$\Downarrow$

Differentiable.

$f(x)=\begin{cases}x^2+1 & \text{if } x>1 \quad ①\\ ax+b & \text{if } x\le 1. \quad ②\end{cases}$

① $'$: $2x\Rightarrow x=1\Rightarrow 2$.

② $'$: $a\Rightarrow x=1\Rightarrow a$.

$\therefore 2=a$. $\therefore 2=2\cdot 1+b$

$2-2=b$

$b=0$.

$\Rightarrow$ Differentiable: $\lim_{x\to 1^+}=\lim_{x\to 1^-}$

$\because \lim_{x\to 1^+}=$ Derivative of first function.

$\lim_{x\to 1^-}=$ Derivative of see function.

$\because$ pass (1,2)

functions's derivative.

⇓

plug in the x

⇓

The slope of that point

or

The tangent line to this function that passthrough this point.

Eg. $f(x)=\begin{cases} cx^2+4x+1, & x\geq 1 \\ ax+b, & x<1 \end{cases}$.

Find a, b in terms of c such the f(x) is differentiable

∵ Differentiable.

$\therefore \lim_{x\to 1^+} = \lim_{x\to 1^-}$

$\therefore \underbrace{\lim_{x\to 1^+} cx^2+4x+1}_{2c\cdot x+4.(x=1)} = \underbrace{\lim_{x\to 1^-} ax+b}_{a(x=1)}$.

$= \underbrace{2c+4}_{}$. $\quad = \underbrace{a}_{}$.

$\therefore \boxed{2c+4=a}$.

∵ If differentiable, then its also continuous.

∵ cx^2+4x+1 when $x=1$ is $c+4+1$

$\therefore 2c+4+b=c+4+1$

$b=1-c$.

Eg. $f(x)=\begin{cases} ax+b, & x>0 \\ \sin x, & x\leq 0 \end{cases}$. Find the domain which f(x) is continuous but not differentiable

Step 1: Continuous.

∵ $\sin x$, where $x=0$ is $\sin 0=0$.

∴ $ax+b$ pass $(0,0) \Rightarrow b=0$.

∴ For now a is $(-\infty,\infty)$ and $b=0$.

Step 2: Not differentiable.

∵ Differentiable is $\lim_{x\to 0^+} = \lim_{x\to 0^-}$

$\therefore \lim_{x\to 0^+} ax+b = \lim_{x\to 0^-} \sin x$.

$\underbrace{a(x=0)}_{} \quad \underbrace{\cos x(x=0)}_{}$

$= \underbrace{a}_{} \quad = \underbrace{1}_{}$

∴ $a=1$ then DIFFERENTIABLE.

∴ $a\neq 1$

Summary

Inverse Function.

E.g. $y = x^3$ $x = \sqrt[3]{y}$

$\Downarrow$ $\Downarrow$

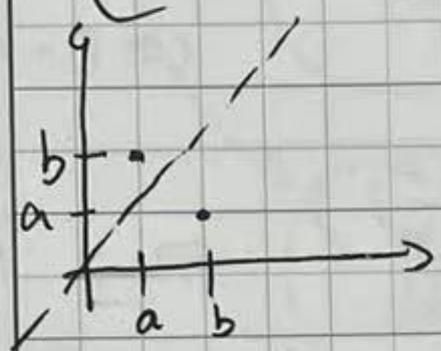
$y = f(x)$ $x = g(y)$.

$x = g(f(x))$ $\Rightarrow$ g undoes f $\quad f = g$ is the inverse of f (or vise versa)

$[g = f^{-1}$ $f = g^{-1}]$.

$f^{-1}(y) = x$.

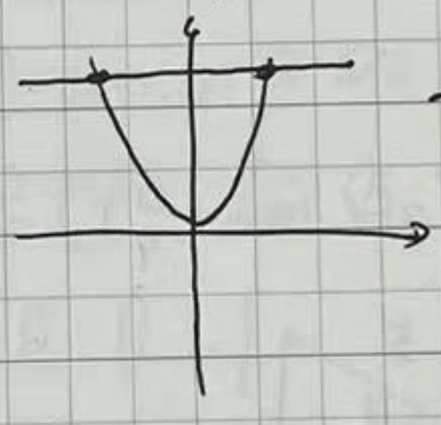
$\Rightarrow$ $f^{-1}(b) = a$ iff. $f(a) = b$.



reflect across $y = x$.

* For function vs VERTICAL LINE TEST.

* When does a function have inverse?



$\Updownarrow$ HORIZONTAL LINE TEST.

$\Downarrow$

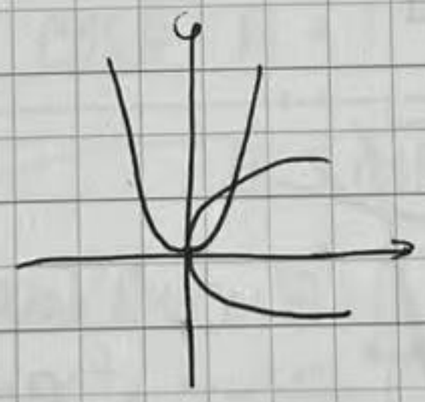
Def: f is one-to-one if, whenever $a \neq b$, $f(a) \neq f(b)$.

How? to tell

① give one y & algebraically get x (number).

How to tell?

② Calculus.



Full ✗.

Partial ✓.

Summary

E.g. ②

For $g(x) = 2x^3 + 3x - 1$.

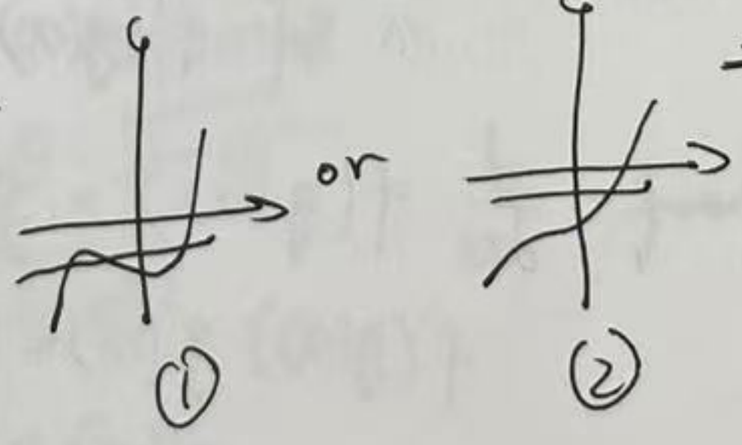
1) $g' = 6x^2 + 3$

2) $\therefore g' > 0$.

3) $\therefore$ Always increasing

4) $\therefore$ ~~Also~~ Is graph ②

Could be



① or ②

range:

1) $\arcsin x = \theta$ in $[-\frac{\pi}{2}, \frac{\pi}{2}]$.

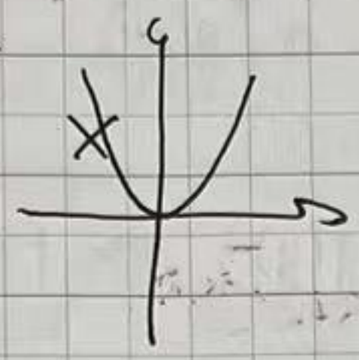
2) $\arccos x = \theta$ in $[0, \pi]$.

3) $\arctan x = \theta$ in $(-\frac{\pi}{2}, \frac{\pi}{2})$

* Partial Inverse.

* Partial has RESTRICTION on <u>original</u> input.

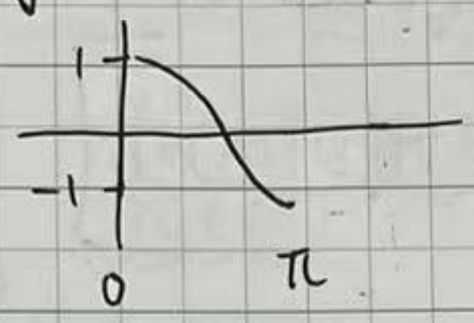
i.e.



$\therefore x \geq 0$.

E.g. $\sqrt{4}$ = num $x \geq 0$ whose square is 4

E.g.



$(\arccos x)$

$\cos^{-1} x$ = angle θ in

$\Rightarrow 0 \leq \theta \leq \pi$.

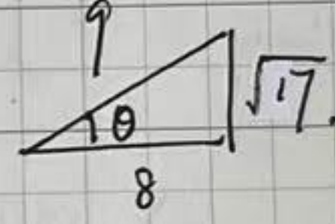
such that

$\cos\theta = x$

E.g. $\tan(\arccos(\frac{8}{9}))$

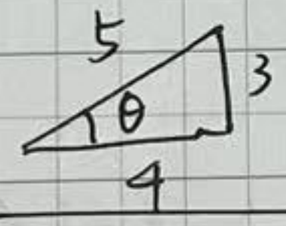
angle θ

$\Rightarrow$

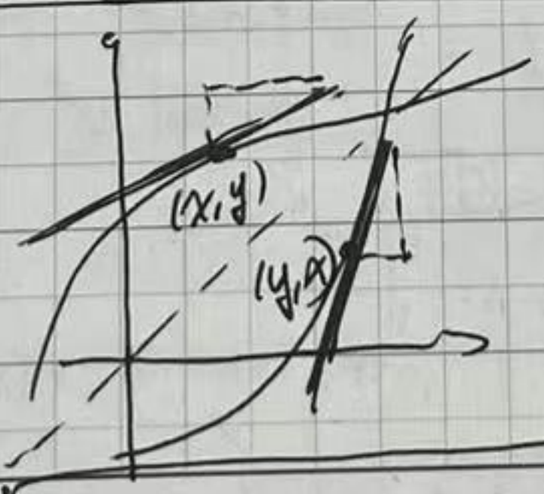


$\therefore \tan(\arccos(\frac{8}{9})) = \frac{\sqrt{17}}{8}$.

E.g. $\sin(\arctan(\frac{3}{4})) = \frac{3}{5}$



* ~~Inverse~~ Derivative of the ~~function~~ inverse functions are reciprocals.



$\therefore$ Two slopes are reciprocal.

Summary

$$g = f^{-1} \Rightarrow g'(x) = \frac{1}{f'(y)} = \frac{1}{f'(g(x))}$$

if $f'(g(x))$ exists & $\neq 0$

[For full proof] Proof: $\frac{d}{dx} f(g(x)) = \frac{d}{dx} x$ [True $\cdot f(g(x)) = x$]

$f'(g(x)) \cdot g'(x) = 1$

$g'(x) = \frac{1}{f'(g(x))}$

Inverse Derivative + Inverse trig.

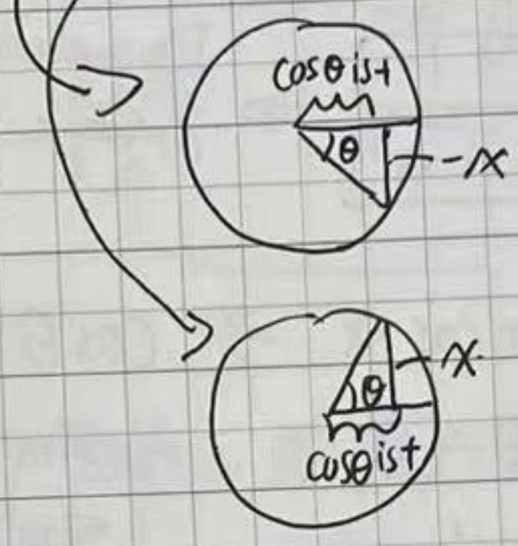
Eg. let $f=\sin x$, $g=\arcsin$.

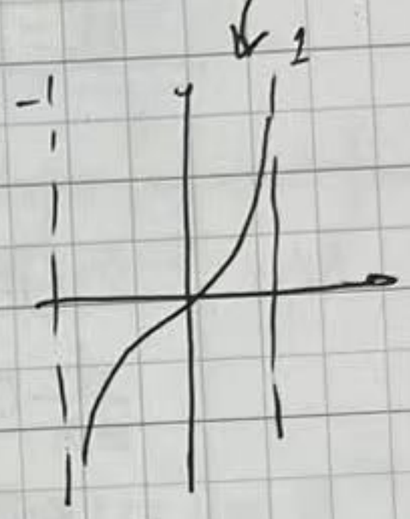
$$\frac{d}{dx}\arcsin(x)=\frac{1}{\sin'(\arcsin x)}=\frac{1}{\cos(\arcsin x)}=\frac{1}{\frac{\sqrt{1-x^2}}{1}}$$

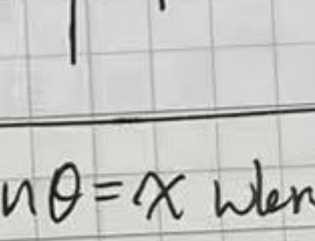
θ

$$=\boxed{\frac{1}{\sqrt{1-x^2}}}.$$

Exists if $-1<x<1$. (which also fits for domain of $\arcsin(x)$)







Where, $\arcsin x$ is partial inverse.

if $x<0$, $-\frac{\pi}{2}<\theta<0$

if $x>0$, $0<\theta<\frac{\pi}{2}$.

Derivative of Inverse + Inverse Trig.

M1.

M2. Implicit way.

E.g. let $\theta=\arcsin x \Rightarrow \sin\theta=x$ where $\theta\in\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$

$\therefore \frac{d}{dx}\sin\theta=\frac{d}{dx}x$.

$\sin'\theta\cdot\theta'=1$

$\cos\theta\cdot\theta'=1$

$\theta'=\frac{1}{\cos\theta}$

* $\sin^2\theta+\cos^2\theta=1$. & $\sin^2\theta=x^2$.

$\therefore x^2+\cos^2\theta=1$.

$\therefore \cos^2\theta=1-x^2$.

$\cos\theta=\pm\sqrt{1-x^2}$.

* set $\theta\in\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$. $\Rightarrow$



$\therefore \cos\theta$ is positive.

$\therefore \cos\theta=\sqrt{1-x^2}$.

$\therefore \theta'=\frac{1}{\sqrt{1-x^2}}$.

Summary

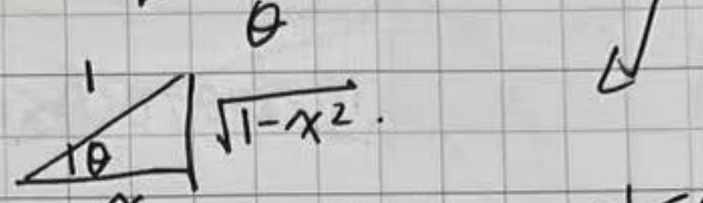
Eg. $\theta = \arccos x$ · find the derivative.

M1: $f = \arccos x \cdot \Rightarrow f' = g$.

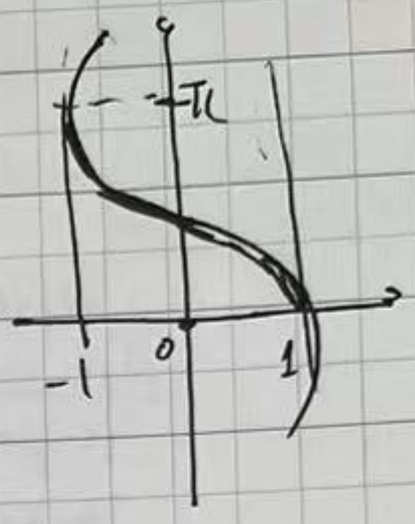
$g = \cos x$.

$\therefore \theta' = \dfrac{1}{\cos(\arccos x)}$

$\theta' = \dfrac{1}{-\sin(\arccos x)}$ ($\arccos x = \theta$)



$\theta' = \boxed{\dfrac{1}{-\sqrt{1-x^2}}}$ $\Rightarrow$ Domain: $-1 < x < 1$ $\Rightarrow$ Range: $[0, \pi]$ $\therefore \arccos : [-1,1] \to [0,\pi]$



M2: $\because \theta = \arccos x \Rightarrow \cos\theta = x$ where $\theta \in [0, \pi]$.

$\dfrac{d}{dx}\cos\theta = \dfrac{d}{dx}x$

$\cos'\theta \cdot \theta' = 1$

$-\sin\theta \cdot \theta' = 1$

$\theta' = \dfrac{1}{-\sin\theta}$

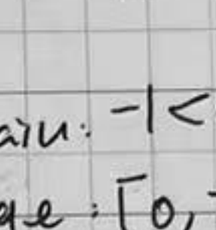
$\because \sin^2\theta + \cos^2\theta = 1$.

$\sin^2\theta\cos^2\theta = x^2$.

$\sin^2\theta = 1 - x^2$.

$\sin\theta = \pm\sqrt{1-x^2}$.

$\because$



$\therefore \sin\theta$ is positive.

$\therefore \sin\theta = \sqrt{1-x^2}$.

$\therefore \theta' = \dfrac{1}{-\sqrt{1-x^2}}$

Eg. Find $\dfrac{d}{dx}\arctan x$ · (Using M2).

$\because \theta = \arctan x \cdot \Rightarrow \tan\theta = x$ where $x \in \mathbb{R}$

$\dfrac{d}{dx}\tan\theta = \dfrac{d}{dx}x$.

$\tan'\theta \cdot \theta' = 1$

$\theta' = \dfrac{1}{\tan'\theta} = \dfrac{1}{\sec^2\theta}$

$\boxed{\theta' = \dfrac{1}{1+x^2}}$

$\because 1 - \tan^2\theta = \dfrac{\cos^2\theta}{\cos^2\theta} - \dfrac{\sin^2\theta}{\cos^2\theta}$

$= \dfrac{\cos^2\theta - \sin^2\theta}{\cos^2\theta}$

$= \dfrac{\cos^2\theta - (1-\cos^2\theta)}{\cos^2\theta} = \dfrac{\cos^2\theta - 1 + \cos^2\theta}{\cos^2\theta}$

$= \dfrac{2\cos^2\theta - 1}{\cos^2\theta} = 2 - \dfrac{1}{\cos^2\theta}$.

$= 2 - \sec^2\theta$

$1 - \tan^2\theta = 2 - \sec^2\theta$.

$\because \tan^2\theta = x^2$.

$1 - x^2 = 2 - \sec^2\theta$

$\sec^2\theta = 2 + x^2 - 1 = 1 + x^2$.

Summary

Derivative of Exponential Function.

$$\frac{d}{dx}a^x = \lim_{\Delta x\to 0}\frac{a^{x+\Delta x}-a^x}{\Delta x} = \frac{a^x\cdot a^{\Delta x}-a^x}{\Delta x} = \frac{a^x(a^{\Delta x}-1)}{\Delta x}.$$

$$= \lim_{\Delta x\to 0} a^x\cdot\frac{a^{\Delta x}-1}{\Delta x} = a^x\cdot\lim_{\Delta x\to 0}\frac{a^{\Delta x}-1}{\Delta x}.$$

Use geometric interpretation to find $\frac{a^{\Delta x}-1}{\Delta x}$.

$\frac{d}{dx}a^x\big|_{x=0}$

$= a^0\cdot\left(\frac{a^{\Delta x}-1}{\Delta x}\right)$.

slope of this function at $x=0$.

$\Rightarrow$ Define $M(e)=1$.

$\therefore \frac{d}{dx}(e^x) = e^x\cdot M(e) = e^x\cdot 1 = e^x$ ☆

$\Rightarrow$ + linear approximation example.

if $e^{4.5}\approx 90$, what is $e^{4.55}$?

$$\frac{e^{4.55}-e^{4.5}}{4.55-4.5} = e^{4.5\prime}$$

$$e^{4.55}-90 = (e^{4.5})(0.05).$$

$$e^{4.55} = 90\times 0.05 + 90.$$

$$= 450 + 90$$

$$\approx 94.5$$

☆ if $\frac{d}{dx}e^{5x}$ [CHAIN RULE!!!].

$\Rightarrow e^{5x\prime}\cdot 5x'$

$= 5e^{5x}$

$M(a) = \ln(a)$ [Proof next pg]

$* \lim_{\Delta x\to 0}\frac{a^{\Delta x}-1}{\Delta x} \text{ be } M(a)$.

$\therefore \frac{d}{dx}a^x = a^x\cdot M(a)$.

Why does e exists.

$f(x) = 2^x$, $f'(0) = M(2)$

⇓ stretch by k (???)

$f(kx) = 2^{kx} = (2^k)^x$

set $2^k = b$

$\therefore f(kx) = b^x$

$\therefore \frac{d}{dx}b^x = b^x\cdot M(2^x)$

$\frac{d}{dx}b^x\big|_{x=0} = b^0\cdot M(0)$

$= M(0)$.

~~$\therefore$ when $b=e$, then $k = M(0) =$~~

Summary

$\therefore \frac{d}{dx}b^x = \frac{d}{dx}f(kx)$

$= f'(kx)\cdot(kx)'$

$= f'(kx)\cdot k$.

$\therefore \frac{d}{dx}b^x\big|_{x=0} = k\cdot f'(kx) = k\cdot M(0)$

$\therefore$ when $b=e$, then $k = \frac{1}{M(2)}$ exists

Review: *

✡ log derivative:

① for e as log base:

$\frac{d}{dx} \ln(x) = \frac{1}{x}$

② for any order:

use i.e. $10^{\log_{10} t} = t$.

then derive.

✡ Or find the derivate of its corresponding exp and then use inverse for derivative

$g'(x) = \frac{1}{f'(g(x))}$ if $g = f^{-1}$

✡ exp derivative:

① for e as base:

$\frac{d}{dx} e^x = (1)(e^x)(1)$

Eg. $\frac{d}{dx} e^{3x} = (3)(e^{3x})(1)$.

② For any base:

$\frac{d}{dx} a^x = \ln(a) \cdot a^x$.

E.g. $\frac{d}{dx} 10^{3x} = (3)(10^{3x})(\ln(10))$

Derivative of logarithms.

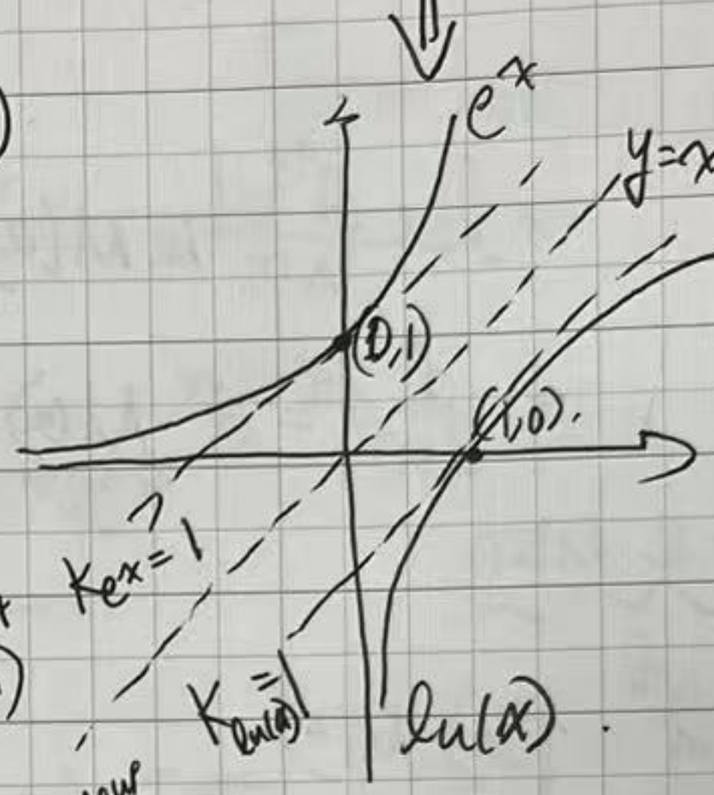
$\log_a(y) = x \Rightarrow a^x = y$.

inverse of a^x → (base)

log base

$\Rightarrow$ $\log(x)$ is inverse of 10^x

$\ln(x)$ is inverse of e^x



$\ln(a_1 a_2) = \ln(a_1) + \ln(a_2)$

↳ $e^{b_1} \cdot e^{b_2} = e^{b_1 + b_2}$.

$\ln(a^n) = n \ln(a)$.

↳ $(e^b)^n = e^{bn}$.

Find Log derivative. $\frac{d}{dx} \ln(x)$:

∵ $\ln(x)$ is the inverse function of e^x

∴ $e^{\ln(x)} = x$., let $\ln(x) = u$.

∴ $e^u = x$.

$\frac{d}{dx} e^u = \frac{d}{dx} x$.

$e^u \cdot u' = 1$.

$u' = \frac{1}{e^u}$

$e^u = e^u$

∴ $u' = \frac{1}{e^u}$

∵ $e^u = e^{\ln(x)} = x$.

∴ $u' = \frac{1}{x}$

$\boxed{\frac{d}{dx} \ln(x) = \frac{1}{x}}$

* For any $\ln(\square)$ *

$\Rightarrow \frac{d}{dx} \ln(\square) = \frac{1}{x}$

(2)

Summary

✡ Exp derivative proof:

$\frac{d}{dx} a^x = M(a) \cdot a^x$

M1: Change base to e

∵ $a = e^{\ln(a)}$

∴ $a^x = (e^{\ln(a)})^x = e^{x \ln(a)}$

∴ $\frac{d}{dx} a^x = \frac{d}{dx} e^{x \ln(a)}$.

$\frac{d}{dx} e^{x \ln(a)} = \ln(a) \cdot e^{x \ln(a)}$.

This is our M(a).

$\Rightarrow \frac{d}{dx} e^{x \ln(a)} = \ln(a) \cdot a^x$

∴ $\frac{d}{dx} a^x = \ln(a) \cdot a^x$

→ 1. Change base to e.

2. Differentiate using chain rule.

(M2.) Logarithmic differentiation → u is a function
→ when diffiant finding $\frac{d}{dx}(u)$ ⇒ use (turn to) $\frac{d}{dx}\ln u$.

$\therefore \frac{d}{dx}\ln u = (\ln u)' \cdot u' = \frac{1}{u}\cdot u' = \frac{1}{u}\frac{du}{dx}$.

$\star (\ln u)' = \frac{u'}{u}$. ←

E.g. for our proof → plug in $u = a^x$.

→ find $\frac{d}{dx}a^x \Rightarrow \ln a^x = x\ln a$.

$\therefore \frac{d}{dx}x\cdot\ln a = \ln a$.

$\therefore \frac{d}{dx}a^x = \left(\frac{d}{dx}x\cdot\ln a\right)\cdot(a^x) = \ln a \cdot a^x$

Eg.2. Find $\frac{d}{dx}(x^x)$.

let $v = x^x \Rightarrow \therefore \ln v = \ln x^x = x\ln x$.

$\therefore \frac{d}{dx}(x\ln x) = \ln x' \cdot x + x'\ln x = \frac{1}{x}\cdot x + \ln x$.

$\therefore \frac{v'}{v} = 1 + \ln x \therefore v' = \frac{d}{dx}(x^x) = (1+\ln x)\cdot x^x$.

$\frac{u'}{u} = \frac{d}{dx}x\ln a = \ln a \Rightarrow$

$\therefore u' = \ln a \cdot a^x$.

---

*Prove log identity:

① let $a = \ln M$, $b = \ln N$.

$\therefore M = e^a$, $N = e^b$

$\ln(MN) = \ln(e^a \cdot e^b)$

$= \ln(e^{a+b})$

$= a+b$.

$\therefore \ln(MN) = \ln M + \ln N$.

② ↳ $\ln\left(\frac{M}{N}\right) = \ln M - \ln N$

③ let $a = \ln M$.

$\therefore M = e^a$.

$\ln(M^k) = \ln(e^{ak})$

$= ak$.

$= \ln M \cdot k$.

$= k\ln M$.

④ let $\log_b M = y$.

$\therefore b^y = M$.

$\therefore \ln(b^y) = \ln(M)$.

$y\ln b = \ln M$.

$y = \frac{\ln M}{\ln b}$.

Log Differentiation technique Eg.:

$y = \sqrt{x(x+4)}$ $(x>0)$, find y'

⇒ ① Find $\frac{y'}{y}$

$\ln y = \ln\sqrt{x(x+4)} = \ln(x(x+4))^{\frac{1}{2}}$

$= \frac{1}{2}\ln(x^2+4x)$

$\therefore \frac{d}{dx}\ln y = \frac{d}{dx}\left(\frac{1}{2}\ln(x^2+4x)\right)$

$= \frac{1}{2}\left(\frac{1}{x^2+4x}\right)\cdot 2x+4$.

$= \frac{x+2}{x^2+4x}$.

$\therefore \frac{y'}{y} = \frac{x+2}{x^2+4x}$, $y' = \frac{(x+2)}{(x^2+4x)}\cdot\sqrt{x(x+4)}$.

Summary

Complete proof of x^r (power rule) using both 1. change e base & 2. logarithmic differentiation.

(1). Change e base.

$x = e^{\log_e x}$

$\therefore x^r = (e^{\log_e x})^r = e^{r\log_e x}$.

$\frac{d}{dx}x^r = \frac{d}{dx}e^{r\log_e x} = e^{r\log_e x}\cdot(r\log_e x)' = e^{r\log_e x}\cdot r\cdot(\log_e x)'$

$= r\cdot e^{r\log_e x}\cdot(\ln x)' = r\cdot e^{r\log_e x}\cdot\frac{1}{x} = r\cdot(e^{\log_e x})^r\cdot\frac{1}{x}$

$= r\cdot x^{-1}\cdot x^r = \underline{r\cdot x^{r-1}}$.

(2) Logarithmic differentiation.

let $x^r = y$.

$\therefore \ln y = \ln x^r = r\cdot\ln x$.

$\therefore \frac{d}{dx}\ln y = \frac{d}{dx}r\cdot\ln x = r\cdot\frac{d}{dx}\ln x = r\cdot\frac{1}{x} = \frac{r}{x}$

$\therefore \frac{y'}{y} = \frac{r}{x}$ $\therefore y' = \frac{r}{x}\cdot y = \frac{r}{x}\cdot x^r = r\cdot\frac{x^r}{x} = \underline{r\cdot x^{r-1}}$.

✱ Any rational or irrational

E.g. Find $f'(x)$ where $f(x) = x^\pi + \pi^x$.

$\Rightarrow f'(x) = \pi x^{\pi-1} + \frac{d}{dx}\pi^x$.

$\Rightarrow$ Use change to e base.

$\therefore \pi = e^{\ln\pi}$

$\therefore \pi^x = e^{x\ln\pi}$.

$\therefore \frac{d}{dx}\pi^x = \frac{d}{dx}e^{x\ln\pi} = e^{x\ln\pi}\cdot(x\ln\pi)' = e^{x\ln\pi}\cdot\ln\pi$.

$= (e^{\ln\pi})^x\cdot\ln\pi = \pi^x\cdot\ln\pi$.

$\underline{f'(x) = \pi x^{\pi-1} + \pi^x\ln\pi}$.

E.g. 2 Find $g'(x)$ where $g(x) = \ln(\cos x)$.

$\therefore g'(x) = \ln'(\cos x)(\cos x)'$

$= \frac{1}{\cos x}\cdot(-\sin x)$

$= \underline{-\tan x}$.

$g(x) = \ln(\cos x)$

$g'(x) = -\tan x$.

E.g. 3 Find $h'(x)$ where $h(x) = \ln e^{(x^2)}$

$\therefore \ln e^{x^2} = x^2\ln e = x^2$

$\therefore h'(x) = \underline{2x}$

E.g. 3 slow way:

$\frac{1}{e^{x^2}}\cdot(e^{x^2})'$

$= \frac{1}{e^{x^2}}\cdot e^{x^2}\cdot 2x$.

$= 2x$.

Summary

Ex 4. $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n$.

1. take ㏑ both sides.

$\therefore \ln\left(1+\frac{1}{n}\right)^n = n\cdot\ln\left(1+\frac{1}{n}\right)$ ⇒ let $n=\frac{1}{\Delta x}$, $\Delta x=\frac{1}{n}$.

$\therefore$ when $n\to\infty$, $\Delta x=\frac{1}{\to\infty}$ $\therefore \Delta x\to 0$.

$\therefore n\cdot\ln\left(1+\frac{1}{n}\right) = \frac{1}{\Delta x}\cdot\ln(1+\Delta x) = \frac{1}{\Delta x}\cdot(\ln(1+\Delta x)-\ln(1))$

$\therefore \lim_{n\to\infty}\ln\left(1+\frac{1}{n}\right)^n = \lim_{\Delta x\to 0}\frac{\ln(1+\Delta x)-\ln(1)}{\Delta x}$. $\because \ln(1)=0$

$\frac{d}{dx}\ln x\Big|_{x=1} = \frac{1}{x}$ where $x=1$ $=$ 1.

$\therefore \lim_{n\to\infty}\ln\left(1+\frac{1}{n}\right)^n = 1$.

$\therefore \lim_{n\to\infty}[\ln\left(1+\frac{1}{n}\right)^n] = \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n$

result to our Qu

$e^{\lim_{n\to\infty}[\ln\left(1+\frac{1}{n}\right)^n]} = e^1 = e$.

↑ same.

$\therefore \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = e$. #.

Summary